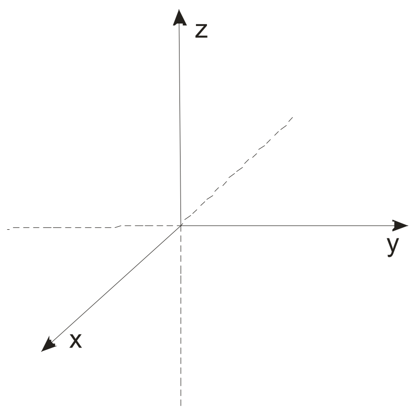
Vectors in space (Part I)

It is best that before you start to study vectors in the area to view the file "Vectors in the plane," because many things are "transferred" from the plane.

Let us first ad a right-angled Cartesian trihedron.

Through a single point set of three vertical lines



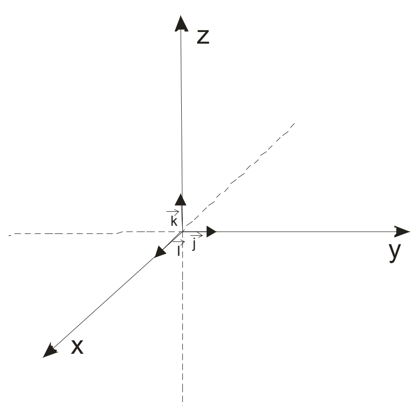
X-axis

Y-axis

Z-axis

This axis make coordinates plane (xOy, xOz and yOz) normal one to another.

On x, y and z observe unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$



$$\vec{i} = (1,0,0)$$

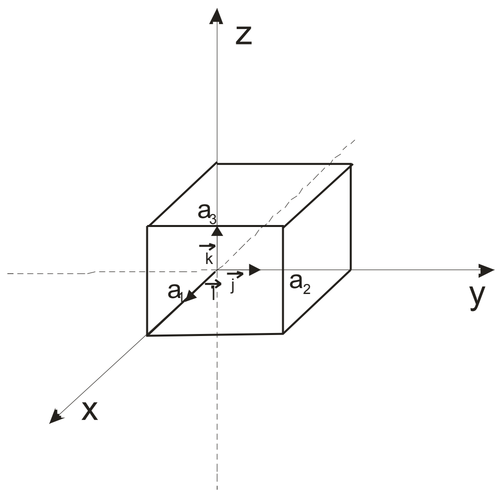
$$\vec{j} = (0,1,0)$$

$$\vec{k} = (0,0,1)$$

$$\left|\vec{i}\right| = \left|\vec{j}\right| = \left|\vec{k}\right| = 1$$

Each vector $\vec{a}$ in the space we present:

$$\boxed{\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \quad \text{or} \quad \vec{a} = (a_1, a_2, a_3)}$$



The intensity vectors $\vec{a}$ is $\left|\vec{a}\right| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Unit vector for vector $\vec{a}$ is vector $\vec{a}_0 = \dfrac{\vec{a}}{\left|\vec{a}\right|}$

If you have two points A and B in space , vector $\vec{AB}$ is :

$$A(x_1, y_1, z_1) \longrightarrow B(x_2, y_2, z_2)$$

$$\vec{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$\left|\vec{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The scalar product **(•)**

If we have vectors

$$\vec{a} = (a_1, a_2, a_3)$$
$$\vec{b} = (b_1, b_2, b_3)$$

Then:

$$\vec{a} \cdot \vec{b} = \left|\vec{a}\right| \cdot \left|\vec{b}\right| \cdot \cos\angle(\vec{a}, \vec{b})$$

If you do not have a given angle between vectors:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Angle between two vectors:

$$\cos\angle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

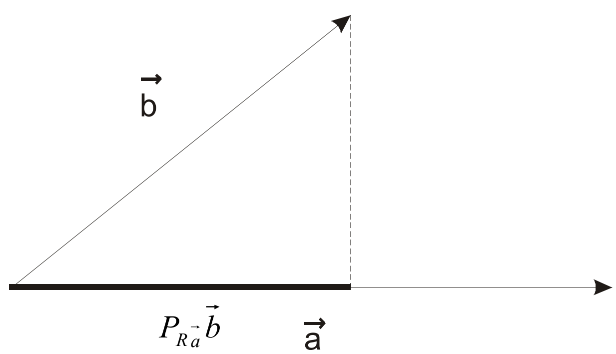
Condition of normality:

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$$

Projection of a vector to another:

$$P_{R_{\vec{b}}}(\vec{a}) = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}$$

$$P_{R_{\vec{a}}}(\vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{a}\right|}$$



Examples:

1) **Find the scalar product of vectors:**

Solution:

$$\vec{a} = (4,-3,1)$$
$$\vec{b} = (5,-2,-3)$$

$$\vec{a} \cdot \vec{b} = (4,-3,1) \cdot (5,-2,-3)$$
$$= 20 + 6(-3) = 23$$

2) **We have vectors** $\vec{a} = (1,-1,2)$ i $\vec{b} = (0,2,1)$. **Determine the angle between vectors** $\vec{a} + \vec{b}$ **and** $\vec{a} - \vec{b}$

Solution:

$$\vec{a} = (1,-1,2)$$
$$\vec{b} = (0,2,1)$$

Find first vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

$$\vec{a} + \vec{b} = (1,-1,2) + (0,2,1) = (1,1,3)$$
$$\vec{a} - \vec{b} = (1,-1,2) - (0,2,1) = (1,-3,1)$$

Mark: $\vec{a} + \vec{b} = \vec{x}$

$$\vec{a} - \vec{b} = \vec{y}$$

So: $\vec{x} = (1,1,3)$ i $\vec{y} = (1,-3,1)$

$$\vec{x} \cdot \vec{y} = (1,1,3) \cdot (1,-3,1) = 1 - 3 + 3 = 1$$
$$\left|\vec{x}\right| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$$
$$\left|\vec{y}\right| = \sqrt{1^2 + (-3)^2 + 1^2} = \sqrt{11}$$

$$\cos \angle(\vec{x}, \vec{y}) = \frac{\vec{x} \cdot \vec{y}}{\left|\vec{x}\right| \left|\vec{y}\right|} = \frac{1}{\sqrt{11} \cdot \sqrt{11}}$$

$$\cos \angle(\vec{x}, \vec{y}) = \frac{1}{11}$$

$$\angle(\vec{x}, \vec{y}) = \arccos \frac{1}{11}$$

3) Find projection of vector $\vec{a}=(5,2,5)$ on vector $\vec{b}=(2,-1,2)$

$$P_{R_{\vec{b}}}(\vec{a})=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|}$$

$$P_{R_{\vec{b}}}(\vec{a})=\frac{18}{3}$$

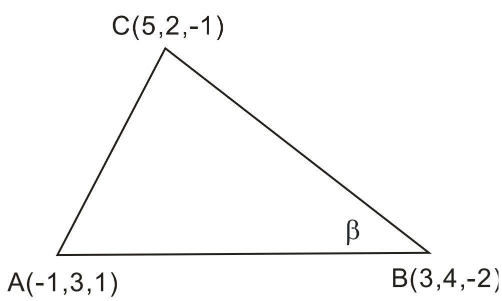
$$P_{R_{\vec{b}}}(\vec{a})=6$$

$$\vec{a}\cdot\vec{b}=(5,2,5)\cdot(2,-1,2)=10-2+10=18$$

$$\left|\vec{b}\right|=\sqrt{2^2+(-1)^2+2^2}=\sqrt{9}=3$$

given the coordinates of the vertices of a triangle

4) Given coordinates of the vertices of a triangle ABC are A(-1,3,1),B(3,4,-2), C(5,2,-1). Set angle ABC.



Find first vectors $\vec{BA}$ and $\vec{BC}$

$$\cos\beta=\frac{\vec{BA}\cdot\vec{BC}}{\left|\vec{BA}\right|\cdot\left|\vec{BC}\right|}$$

$$\vec{BA}=(-1,3,1)-(3,4,-2)=(-4,-1,3)$$

$$\vec{BC}=(5,2,-1)-(3,4,-2)=(2,-2,1)$$

$$\left|\vec{BA}\right|=\sqrt{(4)^2+(1)^2+3^2}=\sqrt{9+1+16}=\sqrt{26}$$

$$\left|\vec{BC}\right|=\sqrt{2^2+(-2)^2+1}=\sqrt{9}=3$$

$$\vec{BA}\cdot\vec{BC}=(-4,-1,3)\cdot(2,-2,1)=-8+2+3=-3$$

$$\cos\beta=\frac{-3}{3\cdot\sqrt{26}}$$

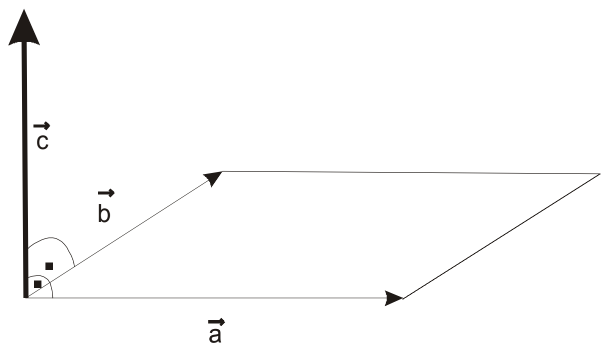
$$\cos\beta=-\frac{1}{\sqrt{26}}$$

$$\beta=\arccos\left(-\frac{1}{\sqrt{26}}\right)$$

Vector product (x)- $\vec{a}\times\vec{b}$

If we have :

$$\vec{a}=(a_1,a_2,a_3)=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$$
$$\vec{b}=(b_1,b_2,b_3)=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$$



Then $\vec{a}\times\vec{b}=\vec{c}$ is vector product Take heed: $\vec{b}\times\vec{a}=-\vec{c}$

1 Vector $\vec{c}$ is perpendicular to the vector $\vec{a}$ and the vector $\vec{b}$

2) The intensity of vector $\vec{c}$, $\left|\vec{c}\right|$ is numerically equal to the surface of the parallelogram over vectors $\vec{a}$ and $\vec{b}$

3) $\vec{c}$ vector is determined by policy of right triedra

The intensity of vector $\vec{a}\times\vec{b}$ is: $\left|\vec{a}\times\vec{b}\right|=\left|\vec{c}\right|=\left|\vec{a}\right|\left|\vec{b}\right|\sin\angle(\vec{a},\vec{b})$

Vectors $\vec{a}$ *and* $\vec{b}$ are collinear if and only if their vector product is equal to 0.

Specifically:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}=\text{ Develop this determined and (for example) get }=$$

$$=\ \#\vec{i}+\$\vec{j}+\&\vec{k}$$

Where #, $, & are some numbers.

Then $\boxed{\left|\vec{a}\times\vec{b}\right|=\sqrt{\#^2+\$^2+\&^2}}$

Parallelogram area over vectors $\vec{a}$ and $\vec{b}$ is P = $\left|\vec{a}\times\vec{b}\right|$

While calculate the area of a triangle (logical) as half of parallelogram area:

$$P_{\Delta} = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

5. Calculate the area of the parallelogram constructed over vectors:

$\vec{a}$ =(1,1,-1) and $\vec{b}$ (2,-1,2)

Solution: P = $\left|\vec{a}\times\vec{b}\right|$

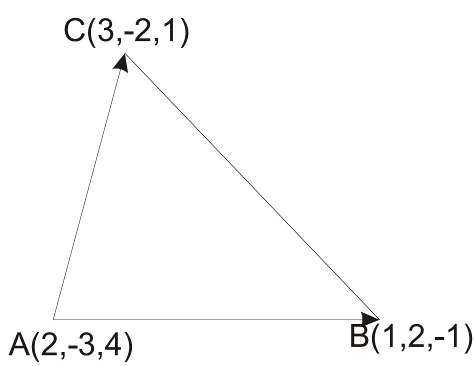
First we must find $\vec{a}\times\vec{b}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix} = \vec{i}\,(2\text{-}1) - \vec{j}\,(2\text{+}2) + \vec{k}\,(\text{-}1\text{-}2) = 1\,\vec{i} - 4\vec{j} - 3\vec{k} = (1, -4, -3)$$

$\left|\vec{a}\times\vec{b}\right| = \sqrt{1^2+(-4)^2+(-3)^2} = \sqrt{26}$ then P= $\sqrt{26}$

6) Calculate the area of a triangle if the coordinates of vertices are: A(2, -3, 4), B(1,2,-1), C(3,-2,1)

Solution: First, create vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$



$\overrightarrow{AB}$ = (1 – 2, 2-(-3), -1 – 4) = (-1,5,-5)

$\overrightarrow{AC}$ = (3-2, -2 – (-3), 1 – 4) = (1,1, -3)

$$P_{\Delta} = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 5 & -5 \\ 1 & 1 & -3 \end{vmatrix} = -10\,\vec{i} - 8\,\vec{j} - 6\,\vec{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-10)^2 + (-8)^2 + (-6)^2} = \sqrt{200} = 10\sqrt{2}$$

$P_\Delta = \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = \frac{1}{2}\,10\sqrt{2} = 5\sqrt{2}$ and solution is here!